NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

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The motion of different bodies immersed in liquid or gaseous media is accompanied by characteristic sound which is excited by the formation of unstable surfaces of separation behind the body, usually disintegrating into a system of discrete vortices (such as the Kármán vortex street due to the flow about an infinitely long rod, etc.). In the noise from fans, pumps, and similar machinery, vortex noise frequently predominates.

The purpose of this work is to elucidate certain questions of the dependence of this sound upon the aerodynamic parameters and the tip speed of the rotating rods or blades. Although some material is given below, insufficient to calculate the first rough approximation to the solution of this question, such as the mechanics of vortex formation (not to speak of the formation of sound), nevertheless certain conclusions may be found of practical application for the reduction of noise from rotating blades.

1. THE FREQUENCIES OF VORTEX SOUND

The dependence of the frequencies of vortex sound from cylindrical rods in parallel flow upon their diameters, length, material, and the flow velocity was first investigated by Strouhal (reference 1). He found that the frequency depends only on the velocity and on the diameter; there is a nondimensional parameter (hereafter called by his name) which takes the constant value

$$(Sh) = \frac{fh_o}{V} = 0.185 \qquad (1)$$

where

f frequency in Hertz

h_o diameter of rod in centimeters

V velocity of flow in centimeters per second

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Following Strouhal, many investigators occupied themselves with a study of the frequency of the oscillation following in the wake of a body (see reference 2 for a list of references). It was found (reference 3) that the Strouhal number varies within sufficiently narrow limits; from 0.09 (for bodies with relatively small lengths) to 0.20 (for infinitely long cylinders), and it is of practical importance that for rotating machines with blades, the Reynolds number range stays constant. Kármán (reference 4) determined the constant (Sh) as 0.194 theoretically. For wing profiles, Lehnert (reference 5) established that the Strouhal number increases somewhat for decreasing angle of attack and for increasing Reynolds number.

The frequency spectrum of the vortex sound from rotating rods was first determined by Stowell and Deming (reference 6). They found experimentally that each element of the rod yielded radiation with the frequency determined by formula (1); that is, that the frequency spectrum was continuous and was within the limits corresponding to the velocities at the beginning and end of the rods. Higher harmonics in the vortex sound were weakly represented.

2. ON THE ORIGIN OF VORTEX SOUND AND ITS ACOUSTICAL POWER

The tests of Stowell and Deming showed that the directional characteristics of vortex sound correspond to those from a dipole with an axis parallel to the axis of rotation; that is, the amplitude of the sound pressure p at a point a sufficient distance r from the source, at an angle δ with the axis of rotation, is determined by the formula

$$p = p_m \cos \delta$$

where p_m is the amplitude of the sound pressure at a distance r on the axis. The tests of E. A. Nepomniyashchi (reference 7) corroborated this fact.

Rayleigh (reference 8) turned attention to the fact that during the flow of air past a wire, oscillations are generated in a direction perpendicular to the direction of flow. This circumstance, as well as the character of the radiation, obliges us to suppose that the origin of vortex sound lies in variable forces acting on the medium during the flow past a body.[1] The sound appears only with the generation of vortices. An already existing ideal vortex street cannot be the origin of sound, because:

(a) The field of flow about a vortex street satisfies Laplace's equation, but not the wave equation.

(b) The vortex street does not radiate acoustical energy; its oscillations in velocity and pressure are "wattless" and are very quickly damped precisely in the direction of the greatest sound radiation.

(c) If a part of the vortex street is isolated by a closed curve moving with the induced velocity, then the flow of energy across this closed curve will be equal to zero and consequently the vortex street cannot radiate sound energy.

(d) A vortex breaking away from a body in an ideal fluid does not have any further effect upon the medium. This applies not only to vortices originating from flow past a body but also to the spiral vortices from propeller blades, and so forth. These vortices therefore cannot be sources of sound. No.! Potential Flow

For the acoustical power in the vortex sound from rotating rods of constant section, Stowell and Deming found experimentally the expression

$$W = \text{const.}\ V^{5.5} L$$

where L is the rod length.

E. A. Nepomniyashchi proposed to apply the laws of similitude of aerodynamic and acoustical quantities in vortex noise. He found theoretically[2] that the acoustical power follows $W = \text{const.}\ V^4$. In this formula no account is taken of the fact that with variable velocity the motion of the air changes the ratio of the wave length

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[1]The effect of the change in a concentrated force F according to a harmonic law with circular frequency ω is equivalent to that from an acoustic dipole of strength $\frac{F}{j\omega\rho}$ (reference 9).

[2]For maintenance of geometrical and kinematic similitude at similar points there is the Euler criterion

$$Eu = \frac{\Delta p}{\rho V^2} = \text{const.};\quad W = \text{const.}\ (\Delta p)^2 = \text{const.}\ V^4$$

See the paper by E. A. Nepomniyashchi (reference 7).

(proportional to velocity) to the dimensions of the radiator (practically constant with velocity change, so that the geometrical flow pattern changes slowly over a wide range of Reynolds number).

One must propose a different formula for the acoustical power in vortex noise to take account of the circumstances indicated.

Let a normal Kármán vortex street be generated behind certain elements of a rod. The complex velocity at an arbitrary point in the complex plane, fixed in relation to the flow, is determined by the Kármán formula

$$V = -\frac{\Gamma_i}{l} \frac{\cosh \frac{\pi h}{l}}{\cos \frac{2\pi z}{l} + i \sinh \frac{\pi h}{l}}$$

where

Γ	absolute value of circulation of an individual vortex
h	distance between the rows of vortices
l	distance between the vortices in a row
z	complex coordinate of a point in fluid

For a stable "chessboard" alinement of a vortex street,

$$\frac{h}{l} = 0.2806 = \text{const.}$$

Potential flow exists for the whole region of z except for the vortex cores. The pressure difference Δp between any two points by Bernoulli's Theorem will be proportional to $\frac{\rho\Gamma^2}{l^2}$ since the expression

$\rho v^2 \approx \rho \frac{\Gamma_i^2}{l^2}$

$$\frac{\cosh \frac{\pi h}{l}}{\cos \frac{2\pi z}{l} + i \sinh \frac{\pi h}{l}}$$

is constant for a fixed z. One may suppose that the amplitude F of a variable harmonic force acting upon the medium and resulting in the appearance of vortex sound, is proportional to the product of the pressure difference Δp and a certain area S', which, in turn, is proportional to a characteristic area of the given body:

$$F = k_1 \Delta p \, S' = k_2 \frac{\rho \Gamma^2}{l^2} S'$$

$\Delta p = \rho V^2 = \rho \left(\frac{\Gamma}{l}\right)^2$

Here $\frac{S'}{l^2} = \bar{S}$, where $\bar{S}$ is a certain constant, the same for all geometrically similar bodies with the same kinematic flow pattern. Then

$$F = k_2 \rho \Gamma^2 \bar{S} \tag{2}$$

The form drag per unit length, according to Kármán, is

$$P = \frac{\rho \Gamma^2}{2\pi l} + \frac{\rho \Gamma (V - 2u) h}{l} \tag{3}$$

where

V the flow velocity at a great distance from the body

u the relative velocity of the vortex street

On the other hand, according to the known formula of experimental aerodynamics,

$$P = C_\alpha \frac{\rho V^2}{2} k_3 h_o \tag{3a}$$

The coefficient C_α represents a coefficient of form drag of the given rod section referred to h_o, the projection of the rod width on the plane perpendicular to the direction of flow. From (3) and (3a) one may find an expression for the circulation

$$\Gamma = \frac{1 - \sqrt{1 - k_4 C_\alpha}}{k_5} hV \qquad (k_4 < 1) \tag{4}$$

$F = K_2 \rho \bar{S} \frac{h^2 V^2}{K_5^2} \left[\cancel{1} - \cancel{1} - K_4 C_\alpha - 2\sqrt{1 - K_4 C_\alpha} \right] = \frac{K_2 K_4}{K_5^2} \rho \bar{S} h^2 V^2 \left[C_\alpha - \frac{2}{K_4} \sqrt{1 - K_4 C_\alpha} \right]$

$F : K_6 \rho \bar{S} h^2 V^2 C_\alpha$

The velocity of a stable vortex street is

$$u = \frac{\Gamma}{2\sqrt{2}\,l}$$

By substituting (4) in (2) and expanding the right hand side in a power series, we obtain an approximation for small C_α:

$$F \cong k_6 C_\alpha \rho V^2 h^2 \overline{S} \qquad (5)^3$$

that is, the amplitude of the force acting on the medium is proportional to the form drag.

As is well known, the acoustical power of a dipole is determined by the formula

$$W = \frac{2}{3}\rho k^4 c\pi A^2 \qquad (6)$$

where

A strength of dipole

k wave number

c velocity of sound

The applied force F is equivalent to a dipole of strength

$$A = \frac{F}{\rho kc}$$

Substitution of this expression in (6) gives

$$W = k_7 k^2 \frac{\rho}{c} h^4 V^4 C_\alpha^{\;2} \overline{S}^2 \qquad (7)$$

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[3] In the opinion of the translator it is believed that C_α in equation (5), and subsequent equations, should be replaced by $C_\alpha^{\;2}$.

The Strouhal number is

$$(Sh) = \frac{fh_o}{V}; \quad k = \frac{2\pi V(Sh)}{ch_o} \tag{8}$$

From (7) and (8) we find

$$W = k_8 \frac{\rho}{c^3} h^2 \left[C_\alpha(Sh)\right]^2 \bar{S}^2 V^6$$

The quantity $h^2\bar{S}$ is proportional to a characteristic area of the given rod, which may be taken as the product of its length L and width h_o. Finally, we obtain

drag coeff. based on frontal area

area of rod

$$W = \text{const.}\ \frac{\rho}{c^3}\left[C_\alpha(Sh)\right]^2 V^6 h_o L \tag{9}$$

speed of sound

This formula applies in the case where the individual rod elements are incoherent radiators. This condition, it seems, is already realized for rotating rods. (V then signifies the tip speed, C_α and (Sh) refer to a certain mean radius.) It is also assumed that the cross-sectional diameter of the rod is small by comparison with the wave length of the radiated vortex sound.

In spite of the qualitative character of the relations obtained, one may draw a series of interesting conclusions:

(1) With C_α and (Sh) independent of velocity (flow for a sufficiently high Reynolds number with a fixed point of breakaway), the acoustical power is proportional to V^6. When this condition is not maintained, the exponent of the power must be less than 6, since for most regimes C_α decreases with increasing velocity; this probably accounts for the 5.5 power observed in the tests of Stowell and Deming.

(2) The smaller the form drag coefficient C_α (or more exactly, the product $C_\alpha(Sh)$), the less the vortex sound.

(3) For increasing rod dimensions, the acoustical power of the vortex sound increases in proportion to the square of the linear dimensions of the rod (for a given tip speed).

3. EXPERIMENTAL VERIFICATION

The vortex noise from rotating rods was studied with equipment installed in a sufficiently large room (8.5 × 18.5 × 7.5 m). The test rods (in two pieces) were fastened to a wooden hub of 350 millimeters diameter, whose shaft was installed in a practically noiseless sleeve bearing. A constant current motor enclosed in a sound-isolating box served to regulate the rotational speed. The connection of the motor with the shaft was by means of a special noiseless Cardan joint.

The sound pressure level in db was measured with an objective General Radio sound meter. The frequency spectrum was determined with the help of a wave analyzer with a quartz filter, made by the same firm. Measurement showed a straight-line frequency characteristic for the sound meter. A piezoelectric microphone was placed exactly on the axis of rotation at a distance of 1000 millimeters from the plane of rotation. Measurement showed that at this distance the predominating quantity is the direct sound, which masks the reflected sound. Sound pressure measurements were repeated at appreciable distances from the source in a diffuse sound field; no qualitative change in the pattern was observed. The background level for all measurements was not less than 10 to 20 db lower than the sound to be measured.

Tests were made with circular rods of different diameters and lengths, with wooden blades having wing profiles (blade profile, the CAHI axial fan series "U" (reference 10), with rectangular aluminum plates of different widths, and with hollow duraluminum rods of egg-shaped section used in aircraft construction. (See fig. 1.) In the latter cases sound measurements were made at different blade angles.

To clarify the effect of the material properties of the rods on the vortex sound, the hollow rods (K1, fig. 1) were tested while filled with cotton-wool (K2) and sand (K3). No differences were observed.

this implies that there was induced flow, thus lift and rotational noise

For the tests of blades and plates with different angles of attack, the microphone was placed on the intake side of the air, outside the wake.

The dependence of the sound pressure level $\left(\text{in db above the threshold of } 10^{-9} \frac{\text{erg}}{\text{cm}^2 \text{ sec}}\right)$ on the tip speed is, for certain rods, given in figure 2. The curves $W = \text{const.}\ V^6$ and $W = \text{const.}\ V^{5.5}$

are drawn on the figure. It is seen from the figure that the vortex sound is more intense the higher the coefficient of form drag and the dimensions of the rods. For flow with a fixed point of breakaway $W = \text{const.} V^6$ but for angles of attack less than the critical there is sufficiently good agreement with the relation $W = \text{const.} V^{5.5}$.

Measurements of the sound pressure level excited by a rotating machine $\left(\text{diameter } 4 \text{ m, tip speed up to } 200 \frac{\text{m}}{\text{sec}}\right)$ from one of the CAHI laboratories provided confirmation of the law $W \sim V^{6 \text{ to } 5.5}$ for high speeds. We also obtained the same results on a series of fans of the axial and centrifugal types.

Figure 3 shows the dependence of the intensity level of vortex sound from blade I2 (fig. 1) on the blade angle. It is seen from the figure that the intensity of the vortex sound increases to correspond with the increase in the form drag coefficient as the blade angle is increased (which in this case gives rise to increase in the angle of attack). Analogous results were obtained for the "U" blades (L1) and for the wide aluminum blades (I2).

Quantitative determination of the acoustical power of the vortex sound and tests to determine the constants entering into formula (9) met with a series of difficulties as a result of a change in the directional characteristics at the commencement of air flow in the axial direction (for blade angle not equal to zero), and of difficulties in the experimental separation of the rotation sound from the vortex sound.

Translated by Dr. E. Z. Stowell
National Advisory Committee
for Aeronautics

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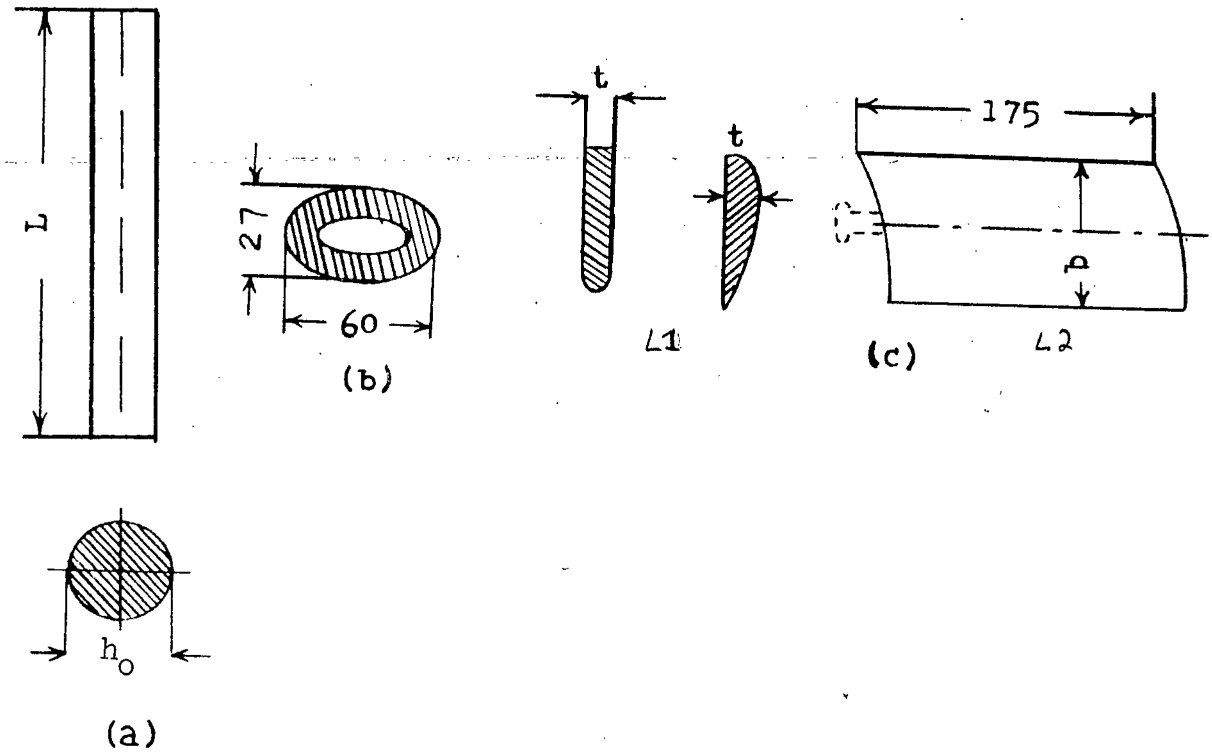

(a) Circular rods

No.	Material	h_o	L
K1	hollow duraluminum tube	32 mm	800 mm
K2	same with cotton-wool IN SIDE TUBE	32	800
K3	same with sand IN HOLLOW TUBE	32	800
K4	wooden rod	15	175
K5	same	58	175

(b) Duraluminum rods with wing profiles
(aviation standard 1 ss)

(c) Blades and plates

No.	Blade type	b	t
L1	Wood, type "U"	87 mm	12 mm
L2	Aluminum plate	65	2

Figure 1.- Test rods, blades and plates.

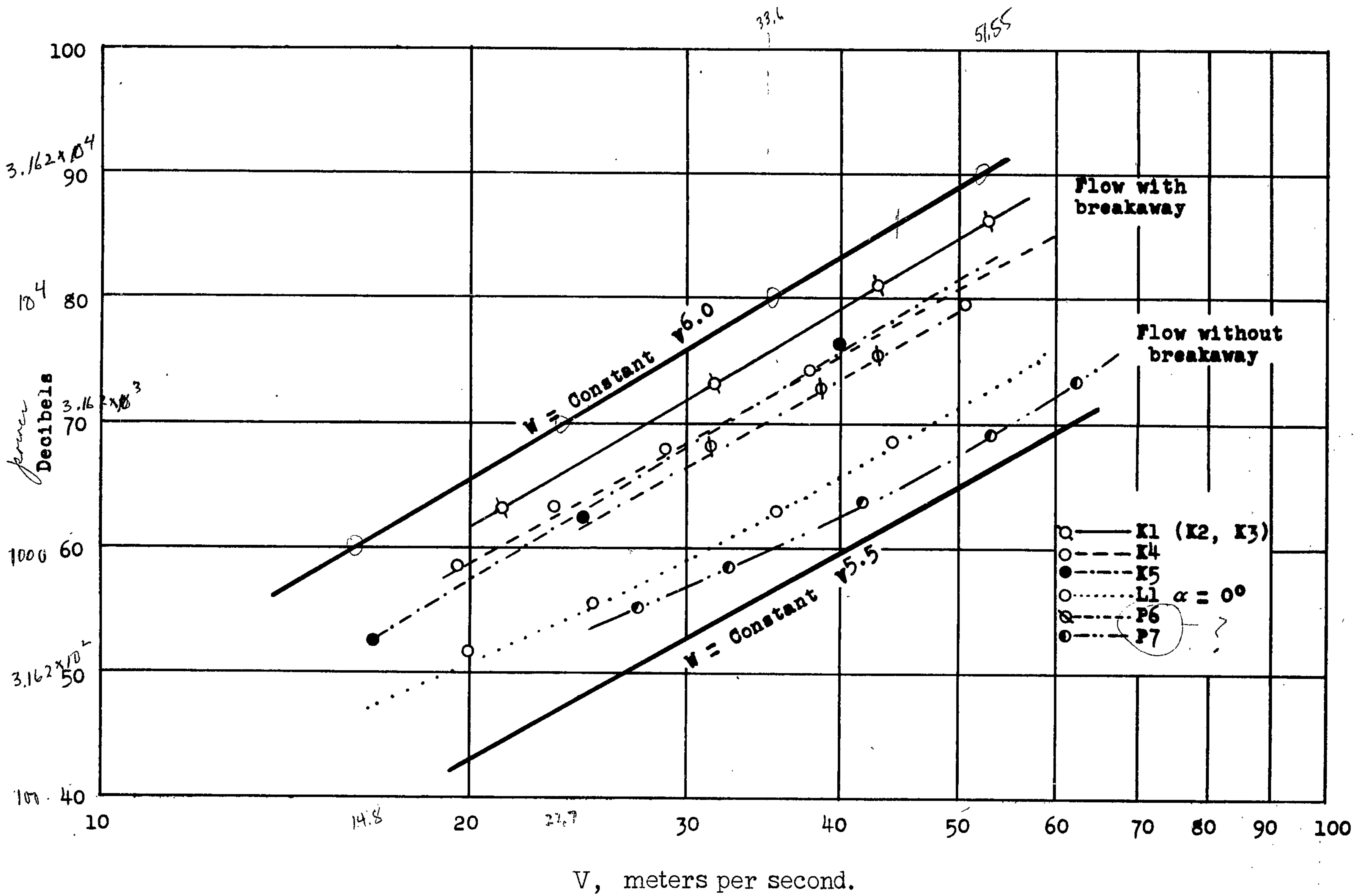

Figure 2.- Dependence of the vortex sound pressure level on tip speed for different rotating rods.

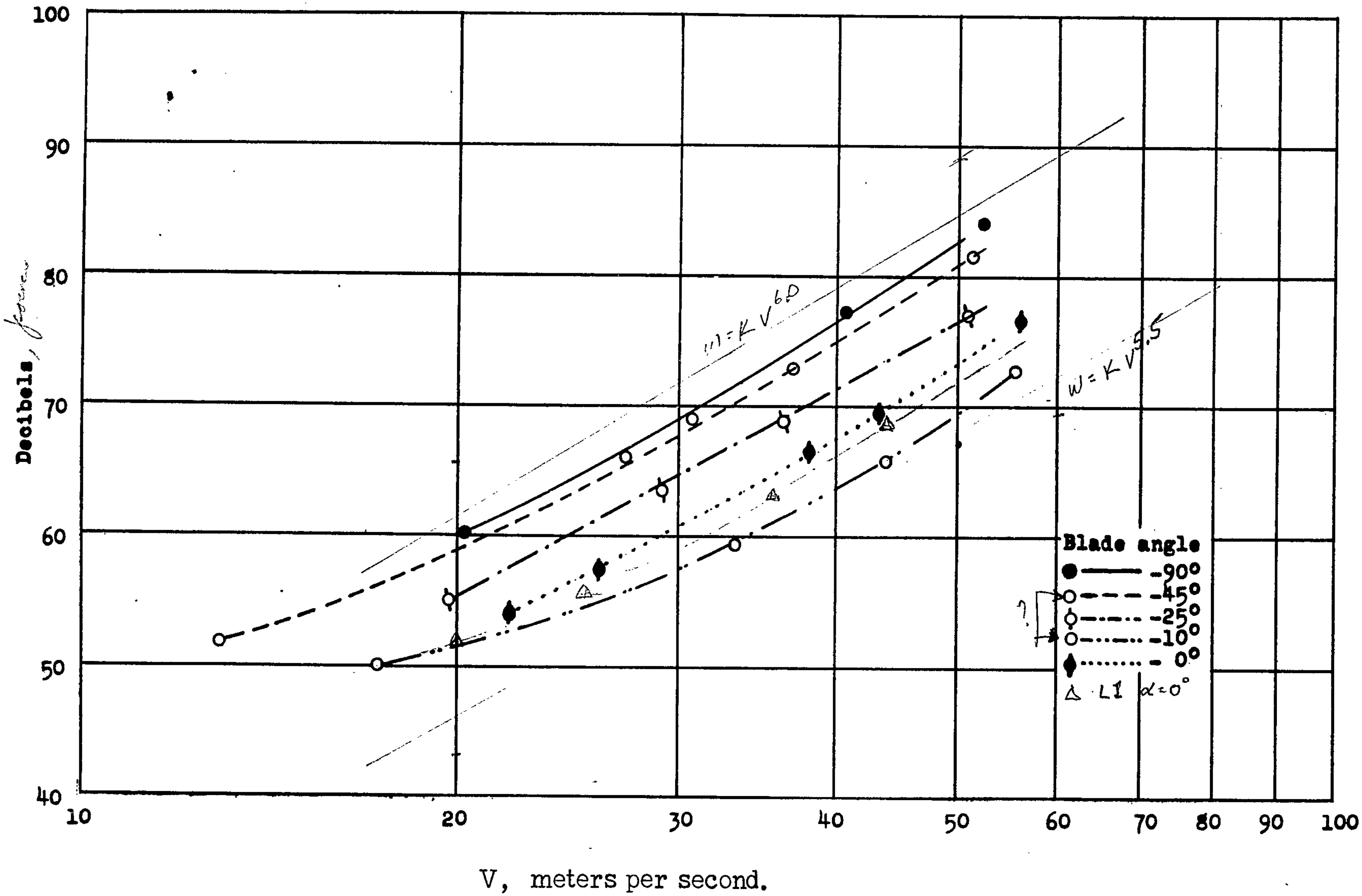

Figure 3.- Dependence of the vortex sound pressure level from rectangular blades L2 on the tip speed and on the blade angle.